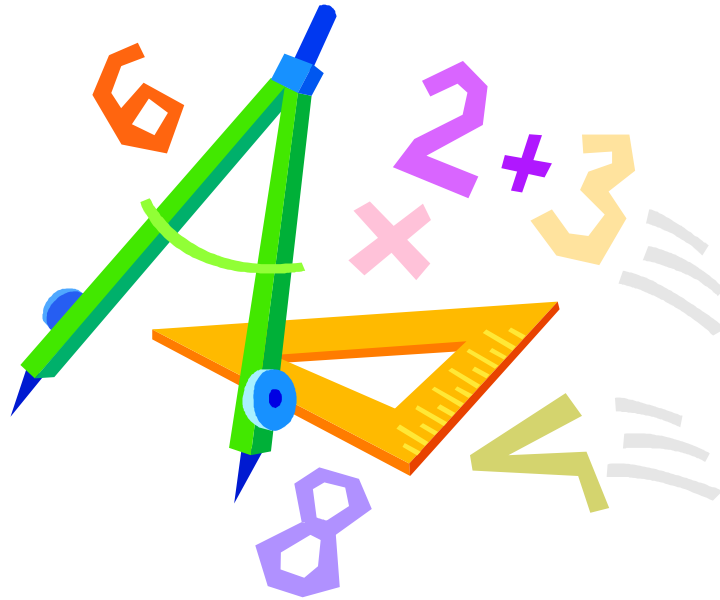


**St Andrew's CEVC Primary School, Great Yeldham.**

# **MATHEMATICS POLICY**



**Revised Autumn Term 2009**

**by Subject Leader Drew Quayle**

**St Andrew's CEVC Primary School**

**Church Road**

**Great Yeldham**

**Halstead**

**Essex**

**C09 4PT**

## **Philosophy**

This policy outlines the teaching, organisation and management of the mathematics taught and learnt at St Andrew's Primary School. The school's policy for mathematics is based on 'The Renewed Framework for teaching mathematics from Foundation Stage to Year 6 (2006).' The policy has full agreement of the staff and Governing Body. The implementation of this policy is the responsibility of all the practitioners in the school learning community.

## **The Nature of Mathematics**

Mathematics is a tool for everyday life. It is a whole network of concepts and relationships which provide a way of viewing and making sense of the world. It is used to analyse and communicate information and ideas and to tackle a range of practical tasks and real life problems. It also provides the materials and means for creating new imaginative worlds to explore.

## **Aims and Objectives**

Using the Programmes of Study from the National Curriculum and the Renewed Framework for Teaching Mathematics (2006) it is our aim to develop:

- A positive and enthusiastic attitude towards mathematics and a fascination of the subject;
- Mathematical understanding through practical tasks, games, enquiry and understanding;
- Competence and confidence in mathematical knowledge, concepts and skills;
- An ability to solve problems, to reason, to think logically and to work systematically and accurately;
- Initiative and an ability to work both independently and in cooperation with others;
- An ability to communicate mathematics;
- An ability to use and apply mathematics across the curriculum and in real life;
- An understanding of mathematics through practical tasks and process of enquiry And experiment;
- And ensure a progressive development of mathematical concepts, knowledge, skills and attitudes.

## **Planning**

All teachers complete long (year), medium (half-termly) and short (daily) term planning. This follows a common format throughout the school. Teachers in Foundation Stage base their teaching on objectives in the Early Years Foundation stage Framework 2007; this ensures that they are working towards the 'Early Learning Goals for Mathematical Development'. In Key Stage 1 and 2 teachers plan for the teaching of mathematics using the New Primary Framework and the National Curriculum. Teachers adapt each mathematical unit to the needs of their class by highlighting the 'building on previous learning' section of each unit. For each lesson, teachers plan specific learning intentions and success criteria based on developing

children's skills, knowledge and understanding in each mathematical area. Where possible teachers make links between subjects to provide experiences that enrich learning and to consolidate and apply the skills that the children have learnt in a variety of contexts.

## **Learning and Teaching**

To provide adequate time for developing mathematical skills each class teacher will provide five hours of mathematics per week. These may vary in length but will usually last between 45 to 60 minutes. Additional mathematics may be taught within other subject lessons when appropriate. In the Foundation Stage mathematics is taught through a range of learning contexts with shorter focused activities. Towards the end of Foundation Stage teachers aim to draw the elements of a daily mathematics lesson together so that by the time children make the transition into Year 1 they are familiar with sustained mathematical sessions.

## **Resources**

All classes have a basic stock of mathematical apparatus and games appropriate to the children's needs. There is also a central store of resources, based in the resource room. School ICT resources include the use of the school laptops which are installed with RM Maths, Number Shark, and the internet.

## **Assessment**

Assessment of pupil work and progress is ongoing by the class teacher and informs future planning. Teachers mark work in mathematics in line with the school marking policy. Teachers use the APP (Assessing Pupil Progress) tool of the Primary Framework to track pupil progress in the focus areas of mathematics agreed each year. APP allows teachers to level children's progress in mathematics, gathering evidence every half term, which is levelled and then input onto Target Tracker. Teachers use this information to inform planning for groups and individual pupils. In the core subjects, statutory assessments are made at the end of Foundation Stage and end of Key Stage 1 and 2. Parent/child/teacher discussions are held each term and are focused around the termly progress report to parents and end of year report.

## **Equal Opportunities**

All teaching and non-teaching staff at St Andrew's Primary School are responsible for ensuring that all children, irrespective of gender, ability, ethnic origin and social circumstances, have access to the whole curriculum and make the greatest possible progress. All children have equal access to the Mathematics Curriculum, its teaching and learning throughout any one year. Day-to-day monitoring of the mathematics policy, and the provision of equal opportunities in Mathematics is the responsibility of the class teacher.

## **Barriers to Learning and Special Educational Needs**

Within the daily mathematics lesson teachers aim to provide activities to support children who find mathematics difficult. Children with barriers to learning are taught within the daily mathematics lesson and are supported to access learning in all lessons. Where applicable children's IEPs include suitable objectives from the Renewed Mathematics Framework and teachers keep these objectives in mind when planning work. When educational support staff, including High Level Support Assistants and Teaching Assistants, are available to support groups or individual children they work collaboratively with the class teacher. The support teacher feeds back to the class teacher when appropriate to inform evaluations, assessment and future planning. Classes are of mixed ability, with overlapping age groups. All children within the school have access to the RM Maths ICT support program. The school also uses a range of intervention programmes with groups and individuals as appropriate, including WAVE 3 GAP pack and Overcoming Barriers to Learning ICT packages.

## **More able, Gifted and Talented**

For the majority of the week the more able children in mathematics will be taught with their own class and extended through differentiated group work and extra challenges. Teachers plan using a 'top down' approach to ensure that the more able are challenged, using problem solving activities that are provided in each class. Differentiation may be by outcome, support, resource or sometimes by the lesson input that is given to different groups by the teacher or a teaching assistant. Children who have been identified as more able, gifted and talented are given opportunities to extend their learning through problem solving, investigation and open-ended activities. We create possibilities for them to work independently and with others to develop higher order thinking skills.

## **Homework**

Homework will be set to each year group in accordance with the school homework policy.

## **Roles and Responsibilities**

### **The Role of the Head teacher:**

The overall responsibility for each subject rests with the senior leadership of the school. The head, in consultation with the staff:

- determines a curriculum that is inclusive to all;
- decides the provision and allocation of resources;
- decides ways in which progress can be assessed, and records maintained;
- ensures that each subject is used in a way to achieve the aims and objectives of the school;
- ensures that there is a subject policy, and identifies a subject leader.

**Role of the Subject Leader:**

- ensure teachers are familiar with the framework and help them to plan lessons;
- lead by example in the way they teach in their own classroom;
- prepare, organise and lead INSET, with the support of the Head teacher;
- work co-operatively with the SENCO;
- observe colleagues, interview pupils, complete book and planning scrutiny from time to time with a view to identifying the support they need;
- attend relevant courses to keep up to date
- organise maths workshops for parents with external body
- discuss termly with the Head teacher and mathematics governor the progress of implementing the Strategy in the school.

**The Role of the Teacher:**

Individual teachers are responsible for the implementation of each subject policy. It is their responsibility to plan appropriate experiences that teach key skills while developing children's knowledge and understanding, using the Renewed Framework. Teachers are responsible for assisting the subject leader in the monitoring and recording of pupil progress in each subject. Individual teachers are expected to undertake training in mathematics as identified.

<b>Calculation Policy</b>
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A further policy entitled Mental and Written Calculation (October 2009), should be used to supplement this mathematics policy.

<b>Evaluation</b>
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The head teacher has overall responsibility for monitoring and evaluating the curriculum, in consultation with the Mathematics Subject Leader and other staff.

Revised by **Drew Quayle** **September 2009**

Shared with staff **September 2009**

Shared with Governors **October 2009**

Next review date **September 2012**

# Mental and Written Calculation Policy

This policy supplements the Mathematics Policy (September 2009).

## Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

**At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.**

**The overall aim is that when children leave primary school they:**

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

## Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of

structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

**Secure mental calculation requires the ability to:**

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year 2), sums and differences of multiples of 10 (Year 3) and multiplication and division facts up to  $10 \times 10$  (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2).
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

**Written methods of calculation**

*The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient method for each operation with confidence and understanding. The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.*

Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

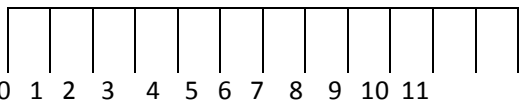
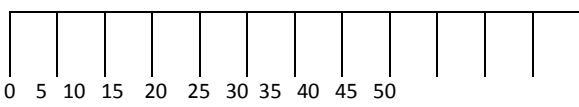
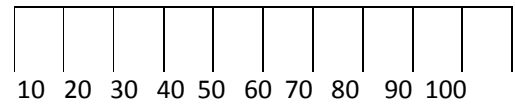
## Written methods for addition of whole numbers




The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and **one** efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

**To add successfully, children need to be able to:**

- recall all addition pairs to  $9 + 9$  and complements in 10, (such as  $\square + 3 = 10$ );
- add mentally a series of one-digit numbers, (such as  $5 + 8 + 4$ );
- add multiples of 10 (such as  $60 + 70$ ) or of 100, (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.**

Progression in use of number line	Number track												
<p>To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to be progression in their use of number tracks and number lines</p>	<p data-bbox="820 835 982 861"><b>Number track</b></p> <table border="1" data-bbox="836 955 1388 1018"><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr></table> <p data-bbox="820 1092 1201 1117"><b>Number line, all numbers labelled</b></p>  <p data-bbox="820 1344 1185 1369"><b>Number line, 5s and 10s labelled</b></p>  <p data-bbox="820 1585 1112 1610"><b>Number lines, 10s labelled</b></p>  <p data-bbox="820 1837 1242 1862"><b>Number lines, marked but unlabelled</b></p>	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12		

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<p><b>Stage 1: The empty number line</b></p> <ul style="list-style-type: none"> <li>The mental methods that lead to column addition generally involve partitioning. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</li> <li>The empty number line helps to record the steps on the way to calculating the total.</li> </ul>	<p><b>Stage 1</b></p> <p>Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.</p> <p><math>8 + 7 = 15</math></p>  <p><math>48 + 36 = 84</math></p>  <p>or:</p> 																				
<p><b>Stage 2: Partitioning</b></p> <ul style="list-style-type: none"> <li>The next stage is to record mental methods using partitioning into tens and ones separately. Add the tens and then the ones to form partial sums and then add these partial sums.</li> <li>Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.</li> </ul>	<p><b>Stage 2</b></p> <p>Record steps in addition using partitioning:</p> <p><math>47 + 76</math>  <math>47 + 70 + 6 = 117</math>  <math>117 + 6 = 123</math></p> <p>or <math>47 + 76</math>  <math>40 + 70 = 110</math>  <math>7 + 6 = 13</math>  <math>110 + 13 = 123</math></p> <p>Partitioned numbers are then written under one another, for example :</p> $\begin{array}{r} 47 = 40 + 7 \\ + 76 \quad \underline{70 + 6} \\ \hline 110 + 13 = 123 \end{array}$																				

<p><b>Stage 3: Expanded method in columns</b></p> <ul style="list-style-type: none"> <li>• Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums initially the tens, not the ones, are added first, following mental methods. The total of the partial sums can be found by adding them together.</li> <li>• The addition of the tens in the calculation <math>47 + 76</math> is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.</li> </ul> <p>As children gain confidence, ask them to start by adding the ones digits first every time.</p> <ul style="list-style-type: none"> <li>• The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.</li> </ul>	<p><b>Stage 3</b></p> <p>Write the numbers in columns.</p> <p>Adding the tens first:</p> $\begin{array}{r} 47 \\ + 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$ <p>Adding the ones first:</p> $\begin{array}{r} 47 \\ + 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$ <p>Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.</p>
<p><b>Stage 4: Compact column method</b></p> <ul style="list-style-type: none"> <li>• In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.</li> <li>• Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.</li> </ul>	<p><b>Stage 4</b></p> $\begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array} \quad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ 11 \end{array}$ <p>Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.</p>

## Written methods for subtraction of whole numbers


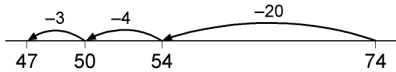
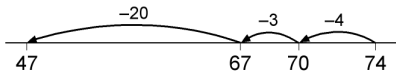
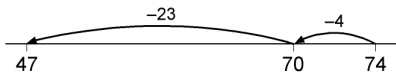
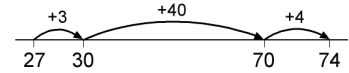
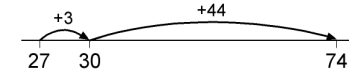
The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and **one** efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

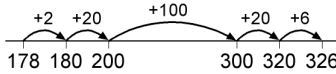
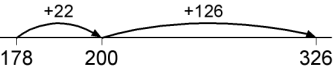
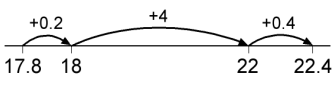
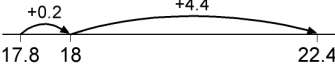
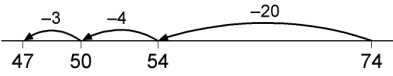
These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers.

**To subtract successfully, children need to be able to:**

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as  $160 - 70$ ) using the related subtraction fact,  $16 - 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into  $70 + 4$  or  $60 + 14$ ).

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.**

<p><b>Stage 1: Using the empty number line</b> <b>Finding an answer by counting back</b></p> <ul style="list-style-type: none"> <li>• The empty number line helps to record or explain the steps in mental subtraction.</li> <li>• A calculation like <math>74 - 27</math> can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.</li> </ul>	<p><b>Stage 1</b> Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. <math>15 - 7 = 8</math></p>  <p><math>74 - 27 = 47</math> worked by counting back:</p>  <p>The steps may be recorded in a different order:</p>  <p>or combined:</p> 
<p><b>Stage 1: Using an empty number line</b> <b>Finding an answer by counting up</b></p> <ul style="list-style-type: none"> <li>• The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeepers method).</li> <li>• <b>With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as <math>57 - 12</math>, <math>86 - 77</math> or <math>43 - 28</math>.</b></li> </ul>	<p><math>74 - 27 =</math></p>  <p>or:</p> 

<ul style="list-style-type: none"> <li>With three-digit numbers the number of steps can again be reduced, enabling children to work out answers to calculations such as <math>326 - 178</math> first in small steps and then more compact by using knowledge of complements to 100</li> <li>The most compact form of recording becomes reasonably efficient.</li> </ul>	<p><math>326 - 178 =</math></p>  <p>or:</p> 
<ul style="list-style-type: none"> <li>The method can successfully be used with decimal numbers.</li> <li><b><i>This method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4 or below.</i></b></li> </ul>	<p><math>22.4 - 17.8 =</math></p>  <p>or:</p> 
<p><b>Stage 2: Partitioning</b></p> <ul style="list-style-type: none"> <li>Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For <math>74 - 27</math> this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn.</li> </ul> <p><b>This use of partitioning is a useful step towards the most commonly used column method, decomposition</b></p>	<p><b>Stage 2</b></p> <p>Subtraction can be recorded using partitioning:</p> <p><math>74 - 27</math>  <math>74 - 20 = 54</math>  <math>54 - 7 = 47</math></p> <p>This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.</p> 
<p><b>Stage 3:</b></p>	
<p><b>Expanded layout, leading to column method (Decomposition)</b></p> <ul style="list-style-type: none"> <li>Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.</li> <li>This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.</li> <li><b>The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.</b></li> </ul>	

Example:  $563 - 241$ , no adjustment or decomposition needed

Expanded method

$$\begin{array}{r} 500 + 60 + 3 \\ - 200 + 40 + 1 \\ \hline 300 + 20 + 2 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example:  $563 - 246$ , adjustment from the tens to the units

$$\begin{array}{r} \phantom{500} \phantom{+} \overset{50}{\cancel{60}} \phantom{+} \overset{13}{\cancel{3}} \\ 500 \phantom{+} \cancel{60} \phantom{+} \cancel{3} \\ - 200 \phantom{+} + 40 \phantom{+} + 6 \\ \hline 300 \phantom{+} + 10 \phantom{+} + 7 \phantom{=} = 317 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, how there is a "snag" with the units and the need to exchange a ten. To release units  $60 + 3$  can be partitioned into  $50 + 13$ . The subtraction of the tens becomes '13 minus 6',

Example:  $563 - 271$ , adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} \overset{400}{\cancel{500}} \phantom{+} \overset{160}{\cancel{60}} \phantom{+} + 3 \\ - 200 \phantom{+} + 70 \phantom{+} + 1 \\ \hline 200 \phantom{+} \phantom{90} \phantom{+} \phantom{2} \phantom{=} = 292 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, how there is a "snag" with the tens and the need to exchange a hundred to release needed tens.  $500 + 60$  can be partitioned into  $400 + 160$ . The subtraction of the tens becomes '160 minus 70'.

Example:  $563 - 278$ , adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} \phantom{400} \phantom{+} \overset{150}{\cancel{50}} \phantom{+} \overset{13}{\cancel{3}} \\ - 500 \phantom{+} + \cancel{60} \phantom{+} + \cancel{3} \\ \hline 200 \phantom{+} + 70 \phantom{+} + 8 \\ \hline 200 \phantom{+} + 80 \phantom{+} + 5 \phantom{=} = 285 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how  $60 + 3$  is partitioned into  $50 + 13$ , and then how  $500 + 50$  can be partitioned into  $400 + 150$ , and how this helps when subtracting.

Example:  $503 - 278$ , dealing with zeros when adjusting

$$\begin{array}{r} \phantom{400} \phantom{+} \overset{90}{\cancel{100}} \phantom{+} \overset{13}{\cancel{3}} \\ - 500 \phantom{+} + \cancel{0} \phantom{+} + 3 \\ \hline 200 \phantom{+} + 70 \phantom{+} + 8 \\ \hline 200 \phantom{+} + 20 \phantom{+} + 5 \phantom{=} = 225 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the  $500 + 0$  is partitioned into  $400 + 100$  and then the  $100 + 3$  is partitioned into  $90 + 13$ .

**Please note that, when calculating with numbers close to a multiple of 100 or 1000, it would probably be more efficient to use a mental method or a number line**

**Stage 4: Compact method for three-digit numbers****NB Expanded method needs to be shown alongside compact method**

Example:  $563 - 241$ , no adjustment or decomposition needed

$$\begin{array}{r} 500 + 60 + 3 \\ - 200 + 40 + 1 \\ \hline 300 + 20 + 2 \\ \hline \end{array} \qquad \begin{array}{r} 563 \\ - 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'

Example:  $563 - 246$ , adjustment from the tens to the units

$$\begin{array}{r} \begin{array}{r} 50 \qquad 13 \\ 500 + \cancel{60} + \cancel{3} \\ - 200 + 40 + 6 \\ \hline 300 + 10 + 7 = 317 \end{array} \qquad \begin{array}{r} 51 \\ \cancel{563} \\ \underline{246} \\ 317 \end{array} \end{array}$$

Ensure that children can explain the compact method, referring to the real value of the digits. They need to understand that they are repartitioning the  $60 + 3$  as  $50 + 13$ .

Example:  $563 - 271$ , adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} \begin{array}{r} 400 \qquad 160 \\ \cancel{500} + \cancel{60} + 3 \\ - 200 + 70 + 1 \\ \hline 200 \quad 90 \quad 2 = 292 \end{array} \qquad \begin{array}{r} 41 \\ \cancel{563} \\ \underline{271} \\ 292 \end{array} \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how  $500 + 60$  can be partitioned into  $400 + 160$ . The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Ensure that children are confident to explain how the numbers are repartitioned and why

## Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and

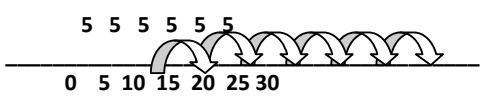
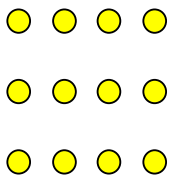
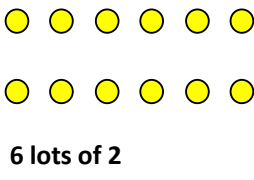


**One** efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

**To multiply successfully, children need to be able to:**

- recall all multiplication facts to  $10 \times 10$ ;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as  $70 \times 5$ ,  $70 \times 50$ ,  $700 \times 5$  or  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

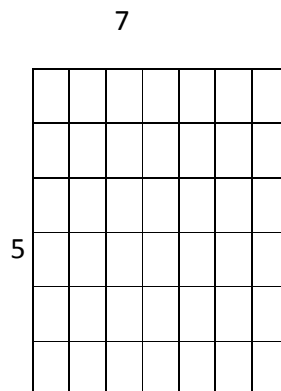
**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.**

### Developing the mental image of multiplication

<p><b>Stage 1 - Number lines</b>          This model illustrates how multiplication relates to repeated addition          Pattern work on a 100 square helps children begin to recognise multiples and rules of divisibility</p>	<p><math>6 \times 5 =</math></p> 
<p><b>Arrays</b>          Successful written methods depend on visualising multiplication as a rectangular array. It also helps children to understand why <math>3 \times 4 = 4 \times 3</math></p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>3 lots of 4</p>  <p>3 X 4</p> </div> <div style="text-align: center;"> <p>6 lots of 2</p>  <p>6 X 2</p> </div> <div style="text-align: center;"> <p>2 lots of 6</p>  <p>2 X 6</p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;"> <div style="text-align: center;"> <p>4 lots of 3</p>  <p>4 X 3</p> </div> </div>	

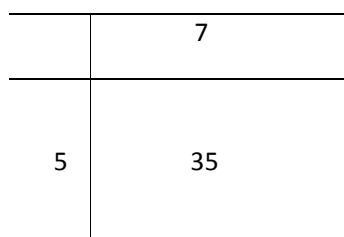
The rectangular array gives a good visual model for multiplication. The area can be found by repeated addition (in this case  $7+7+7+7+7$ )  
 Children should then commit  $7 \times 5$  to memory and know that it is the same as  $5 \times 7$

$7 \times 5 = 35$



Area models like this discourage the use of repeated addition. The focus is on the multiplication facts

$7 \times 5 = 35$



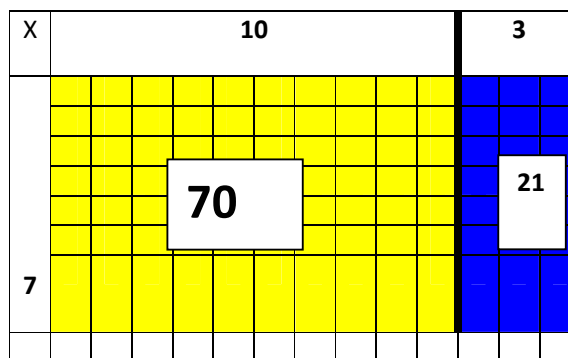
**Stage 2 :** Mental multiplication using arrays and partitioning to multiply a two-digit number by a one-digit number

An array illustrates the distributive law of multiplication i.e.

$13 \times 7$  is the same as  $(10 \times 7) + (3 \times 7)$

Please note that the squares are used to ensure that children have a secure mental image of why the distributive law works

$13 \times 7$



<p>This can lead to the use of a “blank rectangle” to illustrate</p> <p><math>13 \times 7 = (10 \times 7) + (3 \times 7)</math></p> <p><b>Note the rectangle is drawn to emphasise the comparative size of the numbers</b></p>	<p><b>13 X 7</b></p> <p style="text-align: center;"><b>10                  3</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><b>7</b></td> <td style="padding: 5px; text-align: center;"><b>70</b></td> <td style="padding: 5px; text-align: center;"><b>21</b></td> <td style="padding: 5px;"><b>= 91</b></td> </tr> </table>	<b>7</b>	<b>70</b>	<b>21</b>	<b>= 91</b>													
<b>7</b>	<b>70</b>	<b>21</b>	<b>= 91</b>															
<p>Using the grid method to multiply two-digit by one-digit numbers</p> <p>At first children will probably need to partition into 10's (example A)</p> <p>It is important, if they are to use a more compact method, that they can multiply multiples of 10 (example B)</p> <p>i.e. <math>38 \times 7</math> they must be able to calculate <math>30 \times 7</math> as well as <math>8 \times 7</math></p> <p><b>Note the grid is drawn to emphasise the comparative size of the numbers</b></p>	<p><b><math>38 \times 7</math> is approximately <math>40 \times 7 = 280</math></b></p> <p><b>Example A</b></p> <p style="text-align: center;"><b>10      10      10      8</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><b>7</b></td> <td style="padding: 5px; text-align: center;"><b>70</b></td> <td style="padding: 5px; text-align: center;"><b>70</b></td> <td style="padding: 5px; text-align: center;"><b>70</b></td> <td style="padding: 5px; text-align: center;"><b>56</b></td> </tr> </table> <p><b>Example B</b></p> <p style="text-align: center;"><b>30                  8</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><b>7</b></td> <td style="padding: 5px; text-align: center;"><b>210</b></td> <td style="padding: 5px; text-align: center;"><b>56</b></td> <td style="padding: 5px;"><b>=266</b></td> </tr> </table> <p><b>Leading to the layout</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><b>X</b></td> <td style="padding: 5px; text-align: center;"><b>30</b></td> <td style="padding: 5px; text-align: center;"><b>8</b></td> <td></td> </tr> <tr> <td style="padding: 5px;"><b>7</b></td> <td style="padding: 5px; text-align: center;"><b>210</b></td> <td style="padding: 5px; text-align: center;"><b>56</b></td> <td style="padding: 5px;"><b>= 266</b></td> </tr> </table> <p><b>This will lead to a more formalised layout</b></p>	<b>7</b>	<b>70</b>	<b>70</b>	<b>70</b>	<b>56</b>	<b>7</b>	<b>210</b>	<b>56</b>	<b>=266</b>	<b>X</b>	<b>30</b>	<b>8</b>		<b>7</b>	<b>210</b>	<b>56</b>	<b>= 266</b>
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<p><b>Stage 3:</b> Two-digit by two-digit products using the grid method</p> <p>Extend to TU × TU, asking children to estimate first.</p> <p>Start by completing the grid. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product</p> <p><b>Please note that at this stage the grid is no longer drawn to reflect the respective size of the digits. If a child shows signs of insecurity return to rectangular arrays to ensure understanding</b></p>	<p><b>Stage 3</b></p> <p><math>38 \times 14</math> is</p> <table border="1" data-bbox="873 241 1279 504"> <tr> <td>X</td> <td>30</td> <td>8</td> <td></td> </tr> <tr> <td>10</td> <td>300</td> <td>80</td> <td>380</td> </tr> <tr> <td>4</td> <td>120</td> <td>32</td> <td>152</td> </tr> <tr> <td></td> <td></td> <td></td> <td>532</td> </tr> </table> <p>approximately <math>40 \times 15 = 600</math>.</p>	X	30	8		10	300	80	380	4	120	32	152				532				
X	30	8																			
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<p>Three-digit by two-digit products using the grid method</p> <p>Extend to HTU × TU asking children to estimate first.</p> <p><b>Ensure that children can explain why this method works and where the numbers and the grid come from</b></p>	<p><b><math>138 \times 24 =</math> is approximately <math>140 \times 25 = 3500</math></b></p> <table border="1" data-bbox="755 865 1182 1129"> <tr> <td>X</td> <td>100</td> <td>30</td> <td>8</td> <td></td> </tr> <tr> <td>20</td> <td>2000</td> <td>600</td> <td>160</td> <td>2760</td> </tr> <tr> <td>4</td> <td>400</td> <td>120</td> <td>32</td> <td>552</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>3312</td> </tr> </table>	X	100	30	8		20	2000	600	160	2760	4	400	120	32	552					3312
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<p>The grid method works just as satisfactorily with decimal numbers as long as the children can apply their knowledge of multiplication facts to decimal numbers.</p>	<p><b><math>38.5 \times 24</math> is approximately <math>40 \times 25 = 1000</math></b></p> <table border="1" data-bbox="755 1276 1182 1541"> <tr> <td>X</td> <td>30</td> <td>8</td> <td>0.5</td> <td></td> </tr> <tr> <td>20</td> <td>600</td> <td>160</td> <td>10</td> <td>770</td> </tr> <tr> <td>4</td> <td>120</td> <td>32</td> <td>2</td> <td>154</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>924</td> </tr> </table>	X	30	8	0.5		20	600	160	10	770	4	120	32	2	154					924
X	30	8	0.5																		
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<p style="text-align: center;"><b>Schools need to decide if they will teach children the column method.</b></p> <p style="text-align: center;"><b>Schools can decide that the grid is their preferred written method through to level 5</b></p>																					

<p><b>Optional Stage 4 : Expanded short multiplication leading to column method</b></p> <ul style="list-style-type: none"> <li>The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.</li> <li>Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in <math>38 \times 7</math> is 'thirty multiplied by seven', not 'three times seven', although the relationship <math>3 \times 7</math> should be stressed.</li> </ul>	<p><b>Stage 4</b></p> <p><math>38 \times 7</math> is approximately <math>40 \times 7 = 280</math></p> $\begin{array}{r} 30 + 8 \\ \times \quad 7 \\ \hline 210 \quad 30 \times 7 \\ \quad 56 \quad 8 \times 7 \\ \hline 266 \end{array}$ $\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 210 \\ \quad 56 \\ \hline 266 \end{array}$
<p><b>Short multiplication</b></p> <ul style="list-style-type: none"> <li>The recording is reduced further, with carry digits recorded below the line.</li> </ul> <p><b>If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of the grid method</b></p>	<p><math>38 \times 7</math> is approximately <math>40 \times 7 = 280</math></p> $\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 266 \\ \quad 5 \end{array}$ <p>The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.</p>
<ul style="list-style-type: none"> <li>Multiplying two-digit by two-digit numbers includes the working to emphasise the link to the grid method</li> </ul>	<p><math>56 \times 27</math> is approximately <math>60 \times 30 = 1800</math>.</p> $\begin{array}{r} 56 \\ \times \quad 27 \\ \hline 1000 \quad 50 \times 20 = 1000 \\ \quad 120 \quad 6 \times 20 = 120 \\ \quad 350 \quad 50 \times 7 = 350 \\ \quad \quad 42 \quad 6 \times 7 = 42 \\ \hline 1512 \\ \quad 1 \end{array}$
<p><b>Three-digit by two-digit numbers</b></p> <ul style="list-style-type: none"> <li>Continue to show working to link to the grid method.</li> <li>This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive for more confident children to move on to a more compact method.</li> </ul>	$\begin{array}{r} 286 \\ \times \quad 29 \\ \hline 4000 \quad 200 \times 20 = 4000 \\ 1600 \quad 80 \times 20 = 1600 \\ \quad 120 \quad 6 \times 20 = 120 \\ 1800 \quad 200 \times 9 = 1800 \\ \quad 720 \quad 80 \times 9 = 720 \\ \quad \quad 54 \quad 6 \times 9 = 54 \\ \hline 8294 \\ \quad 1 \end{array}$
<p><b>Note most primary school children are unlikely to be ready to use any method more compact by the end of year 6</b></p>	

## Written methods for division of whole numbers

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and

**one** efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.


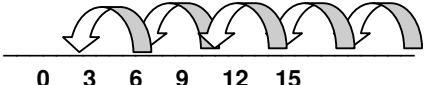
**To divide successfully in their heads, children need to be able to:**


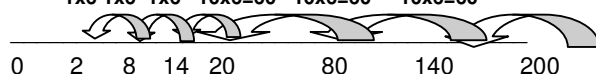

- understand and use the vocabulary of division
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to  $10 \times 10$ , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

**It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.**

**To carry out written methods of division successfully, children also need to be able to:**

- understand division as repeated subtraction (Grouping):
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- Know subtraction facts to 20 and to use this knowledge to subtract multiples of 10 e.g.  $120 - 80$ ,  $320 - 90$

<p><b>Stage 1 Number lines</b></p> <p>Counting on in equal steps based on adding multiples up to the number to be divided</p> <p>Counting back in equal steps based on subtracting multiples from the number to be divided</p> <p><b>Note</b> Counting on is a powerful tool for mental calculation but does not lead onto written calculation for division</p>	<p><math>15 \div 3 =</math></p> <p>+3 +3 +3 +3 +3</p>  <p>- 3 -3 -3 -3 -3</p>  <p>0 3 6 9 12 15</p>
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<p><b>Stage 2 Counting back by chunking</b></p> <p>This method is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.</p> <p>Chunking is useful for reminding children of the link between division and repeated subtraction.</p>	<p><math>100 \div 7 =</math></p> <p><math>4 \times 7 = 28</math>      <math>10 \times 7 = 70</math></p>  <p>Answer 14 remainder 2</p> <p>As you record the division, ask: 'How many sixes in 100?' as well as 'What is 100 divided by 6?'</p>																								
<p>Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.</p> <p><b>Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples</b></p>	<p><math>200 \div 6</math></p> <p><math>1 \times 6 = 6</math> <math>1 \times 6 = 6</math> <math>1 \times 6 = 6</math> <math>10 \times 6 = 60</math> <math>10 \times 6 = 60</math> <math>10 \times 6 = 60</math></p>  <p>Answer 33 remainder 2</p> <p>As you record the division, ask: 'How many sixes in 200?' as well as 'What is 200 divided by 6?'</p> <p>Leading to</p> <p><math>200 \div 6</math></p> <p><math>3 \times 6 = 18</math>      <math>30 \times 6 = 180</math></p> 																								
<p><b>'Expanded' method for TU <math>\div</math> U recorded in columns</b></p> <ul style="list-style-type: none"> <li>This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.</li> <li>As you record the division, ask: 'How many sixes in 90?' or 'What is 90 divided by 6?'</li> <li>This method is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.</li> <li>Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps as illustrated in stage 2, when using a number line</li> </ul>	<p><math>96 \div 6 =</math></p> <p>To find <math>96 \div 6</math>, we start by multiplying 6 by 10, to find that <math>6 \times 10 = 60</math> and <math>6 \times 20 = 120</math>. The multiples of 60 and 120 trap the number 96. This tells us that the answer to <math>96 \div 6</math> is between 60 and 120.</p> <p>Start the division by first subtracting 60 leaving 36, and then subtracting the largest possible multiple of 6, which is 30, leaving no remainder.</p> <table style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td style="text-align: right;">96</td><td></td></tr> <tr><td style="text-align: right;">- 60</td><td>10 X 6</td></tr> <tr><td style="text-align: right;">-----</td><td></td></tr> <tr><td style="text-align: right;">36</td><td></td></tr> <tr><td style="text-align: right;">- 30</td><td>5 X 6</td></tr> <tr><td style="text-align: right;">-----</td><td></td></tr> <tr><td style="text-align: right;">6</td><td></td></tr> <tr><td style="text-align: right;">- 6</td><td>1 X 6</td></tr> <tr><td style="text-align: right;">-----</td><td></td></tr> <tr><td style="text-align: right;">0 16</td><td></td></tr> <tr><td style="text-align: right;">-----</td><td></td></tr> <tr><td style="text-align: right;">Answer 16</td><td></td></tr> </tbody> </table> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin-left: auto; margin-right: auto;"> <p><b>Estimation</b></p> <p>More than <math>10 \times 6 = 60</math> but less than <math>20 \times 6 = 120</math></p> </div>	96		- 60	10 X 6	-----		36		- 30	5 X 6	-----		6		- 6	1 X 6	-----		0 16		-----		Answer 16	
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Answer 16																									

**Stage 3 : 'Expanded' method for HTU ÷ U**

- Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.

The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.

- Estimating has two purposes when doing a division:
  - to help to choose a starting point for the division;
  - to check the answer after the calculation.

To find  $196 \div 6$ , we start by multiplying 6 by 10, 20, 30, to find that  $6 \times 30 = 180$  and  $6 \times 40 = 240$ . The multiples of 180 and 240 trap the number 196. This tells us that the answer to  $196 \div 6$  is between 30 and 40. Initially children will subtract chunks about which they are totally confident. Here a series of chunks ( $6 \times 10$ ) are subtracted to reach 16 then  $6 \times 2$  until no more whole sixes are left, leaving a remainder of 4

196  
- 60  
---  
136  
- 120  
---  
16  
- 12  
---  
4 32

**Estimation**  
More than  
 $30 \times 6 = 180$  but  
less than  
 $40 \times 6 = 240$

Answer 32 R 4

- Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

Here the child has been confident to use the largest possible multiple of 10 as the initial multiplier. Start the division by first subtracting 180 ( $6 \times 30$ ), leaving 16 and then subtracting the largest possible multiple of 6 (which is 12) leaving 4

196  
- 180 30x6  
---  
16  
- 12 2x6  
---  
4 32

**Estimation**  
More than  $30 \times 6 =$   
180 but less than  
 $40 \times 6 = 240$

Answer 32 R 4

The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

**Long division**

The next step is to tackle HTU  $\div$  TU, which for most children will be in Year 6.

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As  $24 \times 20 = 480$  and  $24 \times 30 = 720$ , we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$\begin{array}{r} 24 \overline{) 560} \\ 20 - 480 \\ \hline 80 \\ 3 \quad \underline{72} \\ \hline 8 \end{array}$$

Answer: 23 R 8

Estimation

More than 24  
 $\times 20 = 480$  but

Less than

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$\begin{array}{r} 23 \\ 24 \overline{) 560} \\ \underline{-480} \\ 80 \\ \underline{-72} \\ 8 \end{array}$$

Answer: 23 R 8